



CODESP

The Cooperative Organization for the Development
of
Employee Selection Procedures

PARAEDUCATOR / INSTRUCTIONAL AIDE TUTORIAL

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GENERAL TIPS

APPLICATION PROCESS

To pass the application screening process you must submit a quality application.

- Be thorough. Do not leave spaces blank or questions unanswered. Do not write, "See Resume."
- Do not assume that the screening or human resource staff will know information about you. Always explain in detail, even if you are a current employee or a substitute employee.
- If relevant, include any non-paid volunteer experience, all training, education and paid work experience.
- Promptly submit any name, address, and telephone number changes that may occur during the hiring process. Indicate the exact title of the job for which you have an application on file.
- Make sure you carefully proof all information including spelling, and check for all required signatures. Make sure you correct any errors.
- Make sure you are aware of application deadlines.
- If there is a supplemental application complete it thoroughly and honestly.

TEST PREPARATION

- Before the test: If you have not been to the test center before, check the directions and visit the location before the day of the test.
- Get a good night's sleep. Eat a high-energy breakfast and plan to get to the test site at least 15 minutes early.
- Bring a picture ID.
- If required bring along your invitation to take the test, calculator with charged batteries.
- Bring along a watch and monitor your own time to stay on pace.
- Wear layered clothing so you can adjust to the temperature in the room.

TEN TIPS TO IMPROVE TEST PERFORMANCE

1. Read directions carefully and follow them.
2. Budget your time wisely.
3. Read each question completely.
4. Look for key words.
5. When in doubt, GUESS.
6. Eliminate obvious wrong answers.
7. Check other questions for clues.
8. Answer easy questions first.
9. Don't read too much into a question.
10. Mark your answer sheet properly.

ENGLISH – LANGUAGE ARTS**READING COMPREHENSION**

These questions use a selected passage to test your reading comprehension. To answer these types of questions you must indicate the most appropriate statement relating to the selection on the basis of whether it: 1) accurately paraphrases portions of the selection; 2) adequately summarizes the selection; or 3) presents an inference that can reasonably be drawn from the selection.

SAMPLE**WORK INJURIES**

Use the information below to answer the questions that follow.

The major causes of injuries are slips and falls. Tools, parts, and other objects should not be left lying around. Grease droppings, oils, sludge, and especially polymers should be cleaned up as soon as possible. Warning signs, railings and covers can protect against low piping, open tanks and open manholes or hatches. The simple knowledge of proper lifting techniques, such as bending the knees and lifting using the legs, can save many strained or injured backs.

According to Work Injuries, which one of the following is the primary cause of injury?

- a. improper lifting techniques
- b. grease or polymer burns
- c. slips and falls
- d. low piping

Solution: To answer this question, evaluate each choice.

- Choice **a** lists improper lifting techniques as the primary cause of injury. The paragraph states that only “the simple knowledge of lifting techniques, bending the knees and lifting using the legs, can save many strained or injured backs.” Therefore, this choice is incorrect.
- Choice **b** lists grease or polymer burns as the primary cause of injury. The paragraph states only that “grease droppings, oils, sludge and especially polymers should be cleaned up as soon as possible.” Therefore, this choice is incorrect.
- Choice **c** lists slips and falls as the primary cause of injury. The paragraph states: “The major causes of injuries are slips and falls.” **This choice is correct.**
- Choice **d** lists low piping as the primary cause of injury. The paragraph states that only “warning signs, railings, and covers can protect against low piping, open tanks and open manholes or hatches.” Therefore, this choice is incorrect.

Some reading comprehension questions will ask you to define a target word as it is used in the passage. You must select the most appropriate option that relates to the target word in meaning and usage.

SAMPLE

As used in Work Injuries, PROPER most nearly means

- a. polite
- b. appropriate
- c. incorrect
- d. capable

Solution: To answer this question, evaluate each choice.

- Choice **a** is incorrect. “Polite” is related to the target word, but it cannot be used in the same way that “proper” is used in the passage. Knowledge of *polite* lifting techniques does not mean and cannot be used in the same way as the target word.
- **Choice b is correct.** “Appropriate” is related and can be used like the target word. Knowledge of *appropriate* lifting techniques *can* save many strained or injured backs.
- Choice **c** is incorrect. “Incorrect” is not related to the target word, it means the opposite of “proper,” and cannot be used in the same way.
- Choice **d** is incorrect. “Capable” is related to the target word, but it cannot be used in the same way that “proper” is used in the passage. Knowledge of *capable* lifting techniques cannot be used in the same way as the target word.

MAIN IDEA OF A PARAGRAPH

These questions ask you to first read a paragraph and then choose an answer based on the main idea of the paragraph. The correct answer usually restates the main idea using different wording or requires that you draw a conclusion from the contents of the paragraph.

SAMPLE

A successful weight loss program must contain a specific plan designed to achieve healthy weight loss for an individual. An appropriate plan, without necessary determination to carry it out, is useless. Determination, without a well-defined plan, will only achieve partial success.

The MAIN idea of this paragraph is

- A well-defined plan will assure the success of a weight loss program.
- A high degree of determination is necessary and sufficient for a highly successful weight loss program.
- It’s impossible to develop a successful weight loss program.
- Two important ingredients of a successful weight loss program are a well-defined plan and a sincere resolve to implement that plan.

Solution: To answer this question, evaluate each choice.

- Choice **a** only lists one of the points: a well-defined plan; therefore, this choice is partially correct.
- Choice **b** also only lists one of the points: determination; therefore, this choice is partially correct.
- Choice **c** is not supported by evidence within the paragraph; therefore, this choice is incorrect.
- Choice **d** restates the idea presented in the paragraph. **This choice is correct.**

LOGICAL SEQUENCE OF A PARAGRAPH

These questions ask you to evaluate a paragraph for a smooth, logical progression of ideas. This is known as logical sequencing. First, it is important to know the structure of a paragraph. The **topic sentence** is the first sentence of a paragraph; it introduces the main idea. The **supporting sentences** give details and develop the main idea; they usually follow the topic sentence. The **closing sentence** wraps up the paragraph by restating the main idea, drawing a conclusion, or presenting a transition to another paragraph.

Some questions to ask when evaluating a paragraph’s logical sequence are:

- What is the main idea of the paragraph?
- In what order should the ideas follow?
- Are there ideas that are an extension of the main idea?
- Are there ideas that can’t be understood until other things are explained?

SAMPLE

The following paragraph may not be in a logical sequence. Read the sentences and select the best order for them.

(1) Shoppers are tired of battling crowds at malls and shopping centers. (2) One drawback is the delay in receiving merchandise, but some Internet vendors offer fast shipping for an additional fee. (3) They also enjoy the convenience of being able to shop 24 hours a day, seven days a week. (4) Holiday shopping on the Internet has grown dramatically in the past few years.

- a. 2, 1, 3, 4
- b. 4, 1, 3, 2
- c. 1, 4, 2, 3
- d. correct as written

Solution: To answer this question, evaluate each choice.

- Choice **a** is incorrect because the sentences are not logically ordered: it first presents a supporting sentence (a sentence that is an extension of another idea), then two sentences that do not necessarily relate to the first (but flow with one another), and a closing sentence that is disjointed from the first three sentences.
- Choice **b** presents the sentences in a logical order: a topic sentence, two supporting sentences, and a closing sentence that could transition to another paragraph. **This choice is correct.**
- Choice **c** is incorrect because the sentences are not logically ordered: it presents a topic sentence, then two sentences that relate to the topic and each other, but do not directly develop the topic sentence, and a closing sentence that is disjointed from the first three sentences.
- Choice **d** is incorrect because the sentences are not logically ordered: it presents a topic sentence, then one sentence that does not relate to the first (since Internet shopping hasn't been introduced yet), a third sentence that is misplaced, and a closing sentence that is disjointed from the first three sentences.

ENGLISH GRAMMAR

These types of questions measure your knowledge of grammar, punctuation, capitalization and spelling. You must select the option that has no errors in the areas specified in the question.

SAMPLE

Choose the sentence that shows correct spelling, punctuation and capitalization.

- a. The class, is preparing for their field trip on friday.
- b. The students are exsited to visit the City farm?
- c. Six parent volunteers have agreed to help.
- d. I hope the Spring whether will be nice.

Solution: To answer this question, evaluate each choice.

- Choice **a** is incorrect. The use of the comma after "The class" is incorrect, the word "preparing" is misspelled as "prepairing," and "Friday" should be capitalized.
- Choice **b** is incorrect. The word "excited" is misspelled as "exsited," the word "city" should not be capitalized, and the sentence should end with a period and not a question mark (?).
- **Choice c is correct.** This sentence does not contain any errors in spelling, punctuation, or capitalization.
- Choice **d** is incorrect. The word "spring" should not be capitalized, and the word "weather" is misspelled as "whether."

Some grammar questions will ask you to combine a set of two to three sentences into one sentence with the best grammar. You must select the most appropriate option that summarizes and incorporates all sentences into one sentence with good grammar and structure.

SAMPLE

Which one of the following combines the underlined sentences into a sentence with the BEST grammar?

The students are working on a big project.

The students are busy.

The project is due at the end of the week.

- The students are working, on a project, big and busy, due at the end of the week.
- The students are busy working on a big project that is due at the end of the week.
- The project is due at the end of the week and the students are big and busy.
- The busy students are working on a project that is due at the end of the big week.

Solution: To answer this question, evaluate each choice.

- Choice **a** is incorrect. While all three sentences are combined into one, it is not grammatically correct (has too many commas) and is not summarized correctly (the project is “big,” but not “big and busy”).
- Choice b is correct.** All three sentences are summarized and combined into one structured sentence that is grammatically correct.
- Choice **c** is incorrect. While all three sentences are combined into one, they are not summarized correctly (the students are “busy,” but they are not “big” students).
- Choice **d** is incorrect. While all three sentences are combined into one, they are not summarized correctly or grammatically correct (the project is “big,” but the “week” is not).

WORD USAGE

These types of questions ask you to consider: 1) whether or not a word is used correctly the context of a paragraph; 2) how a word is used in comparison to other sentences; and 3) whether or not a word or phrase is used correctly in a specific sentence. To answer the first type of question (1), it is best to see if the word makes sense in relation to message the sentence/paragraph is communicating. To answer the second type of question (2), it is best to understand what the word in focus means and how it is used to see if it compares with the other sentences offered. To answer the third type of question (3), it is best to select the word or phrase that best fits (or does NOT fit) the sentence, depending on what the question is asking and the main idea of the sentence.

SAMPLE

Find the one word that is used INCORRECTLY in the paragraph below.

The signs of too much stress can show up differently in different people. Such signs can be invisible to you early on (if you look for them). Early detection of stress-related problems usually allows for an easier, more straightforward solution. Being aware of your responses to stress and taking steps toward preventing problems from happening in the first place is even better.

- invisible
- detection
- solution
- preventing

Solution: To answer this question, evaluate each choice.

- **Choice a is correct.** The word “invisible” is used incorrectly because it does not make sense with the rest of the sentence; signs of stress could not be “invisible” if you are looking for them.
- Choice **b** is incorrect. The word “detection” in this sentence makes sense and is used correctly.
- Choice **c** is incorrect. The word “solution” in this sentence makes sense and is used correctly.
- Choice **d** is incorrect. The word “preventing” in this sentence makes sense and is used correctly.

SAMPLE

Which one of the following choices uses the underlined word in the same way it is used in the sentence below?

The primary reason for going on vacation was to take a break.

- The primary school children are very adorable.
- Students learned about mixing primary colors in art class.
- The results of the primary elections are in.
- The primary source of funding came from donations.

Solution: To answer this question, evaluate each choice.

- Choice **a** is incorrect. “Primary” in this sentence refers to the age group of the children in school and it is not used in the same way as the main sentence above.
- Choice **b** is incorrect. “Primary” in this sentence refers to the fundamental colors that can be combined to make a range of colors and is not used in the same way as the main sentence above.
- Choice **c** is incorrect. “Primary” in this sentence refers to elections that narrow down the number of candidates before election for an office. It is not used in the same way as the main sentence above.
- **Choice d is correct.** “Primary” in this sentence refers to the main source of funding and is used in the same way as the main sentence above - it is the main reason for going on vacation.

SAMPLE

Select the word that does NOT correctly complete the sentence.

A good manager ___ and encourages professional development.

- disapproves
- supports
- backs
- promotes

Solution: To answer this question, evaluate each choice.

- **Choice a is correct.** The overall sentence is worded positively and the word “disapproves” does not fit with the rest of the sentence.
- Choice **b** is incorrect. The word “supports” could correctly complete the sentence.
- Choice **c** is incorrect. The word “backs” could correctly complete the sentence.
- Choice **d** is incorrect. The word “promotes” could correctly complete the sentence.

ADDITIONAL TIPS: KEY WORD

In certain multiple choice questions there will be key words that need to be considered when selecting answers. Examples of key words are “best,” “worst,” “first,” “only,” and “never.”

SAMPLE

A child has fallen and is bleeding. The first thing you should do is

- call a doctor
- apply a bandage to the wound
- wash the wound
- apply direct pressure to the wound

Solution: The key word is “first”. Although the other steps may be taken, the **first** step is to control the bleeding by applying direct pressure. Choice **d** is therefore correct.

ADDITIONAL TIPS: BASIC WORD MEANINGS

- A **prefix** is added to the beginning of a word to give the word a new meaning.
- A **suffix** is added to the end of a word to give the word a new meaning.
- By becoming familiar with these prefixes and suffixes you can better answer all types of English questions, from vocabulary to reading comprehension.

Prefix	Meaning	Suffix	Meaning
pre-	before	-ette	small
post-	after	-ess	female
un-	not, opposite	-ize	make
in-	not	-ist	a person who
sub-	under	-ian	one who
inter-	between	-ish	having the quality of
mis-	not or bad	-less	without
dis-	not, opposite	-ous	having
trans-	across	-able	is, can be
anti-	against	-ness	having
pro-	in favor of	-or	one who
sub-	under	-ion	act of
super-	above	-en	made of

Understanding the following terminology will help you determine what the question asks for.

- **Synonyms** are words that are the same in meaning.
- **Antonyms** are words that are opposite in meaning.
- **Verbs** are words that show action.
- **Adverbs** are words that modify verbs.
- **Nouns** are words that name a person, place, thing or idea.
- **Pronouns** are words that take the place of one or more nouns.
- **Adjectives** are words that modify or describe a noun or pronoun. They answer the questions what, which one, how much, and how many?
- **Conjunctions** are words that join together words, phrases, clauses, or sentences.
- **Prepositions** are words that show a relationship between a noun or pronoun and another word in the sentence, such as on, over, from, to, by, through, or between.

GENERAL MATH**KEY WORDS AND CONVERTING WORDS TO EQUATIONS**

Sometimes math questions use key words to indicate what operation to perform. Becoming familiar with these key words will help you determine what the question is asking for.

OPERATION	OTHER WORDS WHICH INDICATE THE OPERATION
Addition	increased by; more than; combined together; total of; sum; added to. The symbol + means add
Subtraction	decreased by; minus; less; difference between/of; less than; fewer than. The symbol - means subtract
Multiplication	of; times; multiplied by; product of (For example: $4 + 4 + 4$ equals 4×3). The symbols \times and \cdot both mean multiply.
Division	per; a; out of; ratio of; quotient of; percent (divide by 100). The symbol \div means divide.
Equal	is; are; was; will be; gives; yields; sold for The symbol = means equal.
Per	divided by
Percent	divide by 100 The symbol % means percent.

Here are some examples of words converted to equations.

WORDS	EQUATIONS
What is the sum of 8 and y?	$8 + y$
4 less than y	$y - 4$
y multiplied by 13	$13y$
The quotient of y and 3	$y / 3$
The difference of 5 and y	$5 - y$
The ratio of 9 more than y to y	$(y + 9) / y$
Nine less than the total of a number (y) and two	$(y + 2) - 9$ or $y - 7$

FRACTIONS

In order to accurately solve fraction problems, it is important to distinguish between the numerator and denominator.

Numerator: top number

Denominator: bottom number

ADDING OR SUBTRACTING FRACTIONS

Adding or subtracting fractions with the same denominator is straightforward.

SAMPLE	$\frac{5}{13} + \frac{6}{13}$
<i>The denominator for both is common so they remain the same. Add the top numbers</i>	$\frac{5 + 6}{13}$
Answer	$\frac{11}{13}$

If you do not have a common denominator (see *SIMPLIFYING FRACTIONS*, below), make one by multiplying the first denominator and the second denominator together.

SAMPLE	$\frac{3}{5} + \frac{2}{7}$
<i>Find the common denominator by multiplying five by seven</i>	$5 \cdot 7 = 35$
<i>To get new numerators, multiply the numerator by the same number as the denominator was multiplied by</i>	$3 \cdot 7 = 21$ $2 \cdot 5 = 10$
<i>Insert the new numbers into the numerator and add the fractions</i>	$\frac{21}{35} + \frac{10}{35}$
Answer	$\frac{31}{35}$

MULTIPLYING FRACTIONS

Multiply the numerator times the numerator and the denominator by the denominator.

SAMPLE	$\frac{1}{4} \cdot \frac{3}{5}$
	$\frac{1 \cdot 3}{4 \cdot 5}$
Answer	$\frac{3}{20}$

Simplify the fraction (see *SIMPLIFYING FRACTIONS*, below) before and after you multiply; this will simplify the problem. (The problem may be calculated without simplifying the fractions, but it will be harder to simplify at the end.)

SAMPLE	$\frac{12}{15} \cdot \frac{5}{6}$
<i>Simplify $\frac{12}{15}$ by dividing both numbers by 3</i>	$\frac{12 \div 3}{15 \div 3} \cdot \frac{5}{6}$
<i>Multiply both numerators and both denominators</i>	$\frac{4}{5} \cdot \frac{5}{6}$
	$\frac{20}{30}$
<i>Simplify by dividing by 10</i>	$\frac{20 \div 10}{30 \div 10}$
Answer	$\frac{2}{3}$

DIVIDING FRACTIONS

Since division is the opposite of multiplication, first invert (flip over) one fraction, then multiply.

SAMPLE	$\frac{1}{5} \div \frac{2}{3}$
<i>Invert $\frac{2}{3}$ and multiply</i>	$\frac{1 \cdot 3}{5 \cdot 2}$
Answer	$\frac{3}{10}$

SIMPLIFYING FRACTIONS

Try dividing both the numerator and the denominator by each prime number.

- Use the rules of divisibility.
- Start with 2: Even numbers (ones that end with 2, 4, 6, 8, or 0) can be divided by two without a remainder.
- Then go to 3: Find the sum of the digits (add the digits together). If the sum can be divided by three, then the number is divisible by 3.
- Next try 5: Numbers that end with 5 or 0 are divisible by five.
- Go on to 7, 11, 13, 17, and so on: Unfortunately, there is no easy way to determine whether the number will be divisible by these – you just have to try dividing by each. But you can stop trying when the number is smaller than the divisor.

SAMPLE	Simplify $\frac{26}{65}$	
<i>Twenty-six can be divided by two without a remainder (because it is even), but 65 can't</i>	$26 \div 2 = 13$	YES
	$65 \div 2 = 32.5$	NO
<i>The digits do not add up to three</i>	$2 + 6 = 8 \div 3$	NO
	$6 + 5 = 11 \div 3$	NO
<i>Sixty-five can be divided by five without a remainder, but 26 can't</i>	$65 \div 5 = 13$	YES
	$26 \div 5 = 5.2$	NO
<i>Try 7</i>	$26 \div 7 = 3.7$	NO
	$65 \div 7 = 9.3$	NO
<i>Try 11</i>	$26 \div 11 = 2.4$	NO
	$65 \div 11 = 5.9$	NO
<i>Try 13 - and it works!</i>	$26 \div 13 = 2$	YES
	$65 \div 13 = 5$	YES
Answer	$\frac{2}{5}$	

WRITING A DECIMAL AS A FRACTION

SAMPLE	Decimal	Fraction	Final Answer
<i>For a number in the tenths place, remove the decimal, divide by ten, and simplify</i>	.5	$\frac{5}{10}$	$\frac{1}{2}$
<i>For a number in the hundredths place, remove the decimal, divide by 100, and simplify</i>	.05	$\frac{5}{100}$	$\frac{1}{20}$
<i>For a number in the thousandths place, remove the decimal, divide by 1,000, and simplify</i>	.005	$\frac{5}{1,000}$	$\frac{1}{200}$

MIXED NUMBERS & IMPROPER FRACTIONS

- A mixed number contains a whole number and a fraction. When the numerator is more than the denominator, it is an improper fraction. Solving equations containing mixed numbers is easier when all mixed numbers are converted to improper fractions.
- A whole number can be converted to an improper fraction by simply making the denominator one.

SAMPLE	$5 = \frac{5}{1}$
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- When working with improper fractions, all the same rules of working with fractions apply

CONVERTING A MIXED NUMBER TO AN IMPROPER FRACTION

- Multiply the whole number by the fraction's denominator
- Add that total to the numerator
- That result is the new numerator and the denominator remains the same.

SAMPLE	Convert to Improper Fraction	$3 \frac{4}{5}$
<i>Multiply the whole number and the fraction's denominator</i>	$3 \cdot 5 = 15$	
<i>Add the numerator</i>	$15 + 4 = 19$	
<i>Put the total above the denominator</i>	$\frac{19}{5}$	
<i>Improper Fraction</i>	$\frac{19}{5}$	

CONVERTING AN IMPROPER FRACTION TO A MIXED NUMBER

- Divide the numerator by the denominator and calculate the whole number including the remainder
- The whole number will be the mixed number's whole number
- The remainder will be the mixed number's numerator

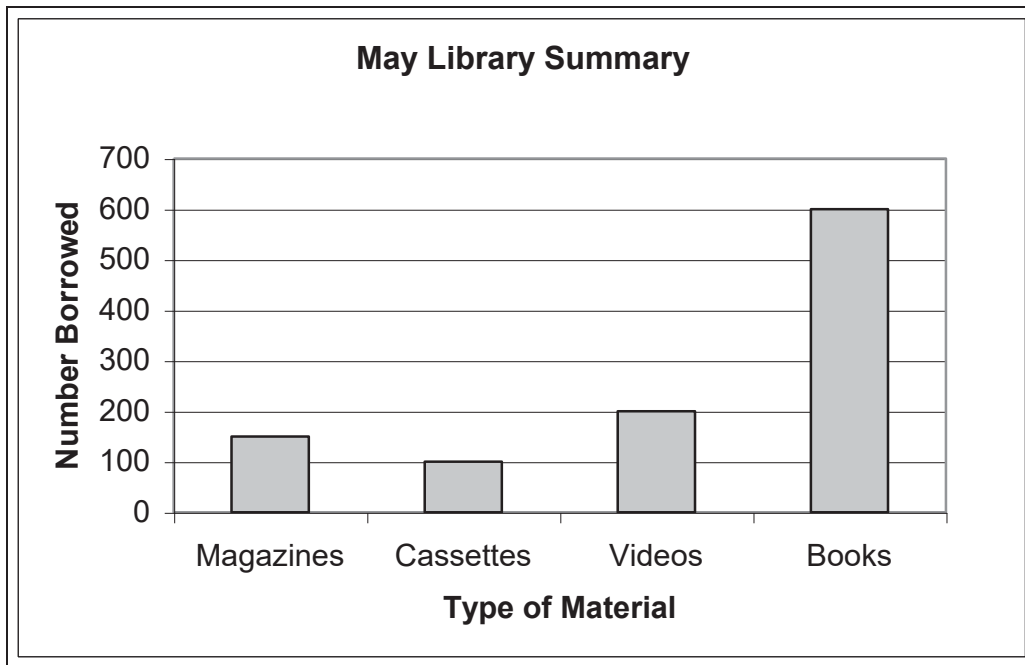
SAMPLE	Convert to Whole Number	$\frac{19}{5}$
<i>Divide the numerator by the denominator</i>	$19 \div 5 = \text{Whole number} = 3$ Remainder = 4	
<i>Use the whole number as the mixed number's whole number and the remainder as the mixed number's numerator</i>	$3 \frac{4}{5}$	
<i>Mixed Number</i>	$3 \frac{4}{5}$	

READING TABLES AND CHARTS

Some questions test the ability to understand, interpret, and use information in tables and charts. Often answering these questions depends on looking in the correct places for information. It is important to know that the horizontal row at the bottom is the x-axis and the vertical column on the left side is the y-axis.

SAMPLES

Use this chart to answer the questions that follow.



According to the May Library Summary chart, what was the number of videos borrowed in May?

- a. 100
- b. 150
- c. 200
- d. 300

Solution: This question requires you to extract information from the chart.

- Begin by determining what information the question asks for.
- Look for the number borrowed (on the left vertical column or y axis) for videos (a type of material found on the horizontal row at the bottom or x axis).
- Find the videos column, then look to the left and read the number that lines up with the top of the videos column.
- Choice **a** is the number of cassettes borrowed in May; therefore, this choice is incorrect.
- Choice **b** is the number of magazines that was borrowed in May; therefore, this choice is incorrect.
- Choice **c** is correct because the top of the “videos” bar meets the line for 200.
- Choice **d** does not correspond to any of the numbers on the chart; therefore, this choice is incorrect.

According to the May Library Summary chart, what percent of the total number of items borrowed were cassettes and videos?

- a. 9.52 %
- b. 19.05 %
- c. 28.57 %
- d. 35.00 %

Solution: This question requires you to extract information from the chart and then calculate a percentage.

- Begin by determining what information the question asks for: cassettes plus videos compared to the total number of materials borrowed.
- Add up the number borrowed in each column to obtain the total ($150 + 100 + 200 + 600 = 1,050$).
- Find the cassettes and videos total ($100 + 200 = 300$).
- Divide the cassettes and videos total by the total number borrowed and multiply by 100 ($300 \div 1,050 = .2857 \cdot 100 = 28.57\%$).
- Choice **a** is only the percentage of cassettes borrowed in May; therefore, this choice is incorrect.
- Choice **b** is only the percentage of videos borrowed in May; therefore, this choice is incorrect.
- Choice **c** is correct because the number of cassettes and videos (300) divided by the total (1,050) and multiplied by 100 is equal to 28.57.
- Choice **d** is only the answer if, in the last step, 1,050 is divided by 300 and then multiplied by 10; therefore this choice is incorrect.

STATISTICS

Find the average (also known as the mean) by adding the sum of the data and dividing that sum by the number of data elements.

SAMPLE	What is the average of 79, 67, 81, 99, 88, and 72?
<i>The six data elements</i>	79, 67, 81, 99, 88, and 72
<i>Add the numbers</i>	$79 + 67 + 81 + 99 + 88 + 72 = 486$
<i>Divide by the number of data elements</i>	$486 \div 6$
Answer	81

EXPONENTS

An exponent is a superscript, or small number written at the top right corner of a number, variable, or parenthesis (for example: 3^4). This tells you to multiply 1 by the number as many times as the exponent says.

SAMPLE	Simplify 3^4
<i>Multiply one by three multiplied by itself four times</i>	$1 (3 \cdot 3 \cdot 3 \cdot 3)$
	$1 (81)$
Answer	81

When multiplying exponents, add the superscripts

SAMPLE	Simplify $x^{16}x^2$
<i>Add the superscripts 16 and 2</i>	x^{16+2}
Answer	x^{18}

When dividing exponents, subtract the superscripts:

SAMPLE	Simplify $\frac{x^6}{x^2}$
<i>Subtract the superscripts ⁶ and ²</i>	x^{6-2}
Answer	x^4

PRE-ALGEBRA AND ALGEBRA

SPECIAL NOTATION FOR MULTIPLICATION AND DIVISION WITH VARIABLES

Here are some examples of special notations and what they mean:

- $2b$ means $2 \cdot b$
- $2(a + 5)$ means twice the sum of a number (a) and five (add a number, represented by the letter a , to 5, then multiply by 2) or $(2 \cdot a) + (2 \cdot 5)$
- bc means $b \cdot c$
- $4bc$ means $4 \cdot b \cdot c$
- $d/5$ means $d \div 5$

ALGEBRA WORD PROBLEMS

In algebra you solve problems by essentially making two groups, one for each side of an equation. An unknown number or value is represented by a letter (for example: x).

BASIC STEPS

- 1) Define the variable
- 2) Translate the problem into an equation and plug in known values
- 3) Set all know values equal to x
- 4) Solve the equation
- 5) Go back to the problem and plug in the new value to obtain the answer

SAMPLE #1	A car dealership has 15 new cars and 12 used cars. How many cars do they have?
<i>Define the unknown variable</i>	Let x = Total Cars
<i>Translate the problem into an equation and insert known values</i>	$15 + 12 = x$
<i>All know values are already equal to x so solve the equation</i>	$27 = x$
Answer	There are 27 Total Cars.

SAMPLE #2	Tickets to the concert are \$20 each. If you spent \$200, how many tickets did you buy?
<i>Define the unknown variable</i>	Let x = The number of tickets you purchased
<i>Translate the problem into an equation and insert known values</i>	$\$20x = \200
<i>Set all known values equal to x by dividing \$20 from both sides of the equation</i>	$\$20x \div \$20 = \$200 \div \20
<i>Setting all known values equal to x has also solved the equation</i>	$x = 10$
Answer	10 tickets were purchased

ORDER OF OPERATIONS

1. Parenthesis and Brackets from the inside out.
2. Exponents of numbers or parenthesis.
3. Multiplication and Division in the order they appear.
4. Addition and Subtraction in the order they appear.

SAMPLE	Simplify the following expression: $2 + (3 - 1)3^2$
<i>Simplify parenthesis & brackets from the inside out (subtract 1 from 3)</i>	$2 + (3 - 1) 3^2 = 2 + (2) 3^2$
<i>Simplify exponents (3^2 becomes 9)</i>	$2 + (2) 3^2 = 2 + (2) 9$
<i>Simplify multiplication and division (multiply 2 and 9)</i>	$2 + (2) 9 = 2 + 18$
<i>Simplify addition and subtraction (combine like terms)</i>	$2 + 18 = 20$
Answer	20

SIMPLIFYING EXPRESSIONS

1. Combine like terms; 2. Simplify multiplication; 3. Distribute a number or sign in to parenthesis;
4. Use the FOIL Method to multiply two or more parenthesis; 5. simplify Exponents of a number

1. Combine Like Terms

- Combine or add up all of the like terms.

Examples of like terms because they are all:

- x with a coefficient
 - 2x, 45x, x, 0x, -26x, -x
- Constants
 - 15, -2, 27, 9043, 0.6
- y^2 with a coefficient
 - $3y^2$, y^2 , $-y^2$, $26y^2$

For comparison, below are a few examples of unlike terms because they:

- are different letter variables
 - 17x, 17z
- are different powers or exponents
 - 15y, $19y^2$, $31y^5$
- both have the letter x but the second term has another variable in it
 - 19x, 14xy

SAMPLE	$5x + 7x$
<i>Add like terms</i>	$5x + 7x = 12x$
Answer	12x

SAMPLE	$14a + 7 + 21a$
<i>Organize like terms together</i>	$14a + 21a + 7$
<i>Add like terms</i>	$14a + 21a + 7 = 35a + 7$
Answer	$35a + 7$

2. Simplify Multiplication

Same Variables

When multiplying same letter variables, keep the letter and add exponents.

SAMPLE	a × a
<i>Neither a has a visible exponent, so their exponents are both 1</i>	$a^1 \cdot a^1$
<i>Add the exponents</i>	$a^{1+1} = a^2$
Answer	a²

Different Variables

SAMPLE	y⁵ · a²
<i>The terms cannot be multiplied by simply adding the exponents because each multiplier is a different letter</i>	$y^5 \cdot a^2$
Answer	y⁵a²

SAMPLE	a² · a³y²
<i>Add the exponents of a² and a³</i>	$a^5 \cdot y^2$
Answer	a⁵y²

3. Distribute a Number or Sign in to Parenthesis

SAMPLE	6 (2 + 4a)
<i>Remove parentheses and multiply each term by six</i>	$(6 \cdot 2) + (6 \cdot 4a)$
Answer	12 + 24a

4. Use the FOIL Method to multiply two or more parenthesis

- First; Outer; Inner; Last

SAMPLE	(3 + 7x) (6 + 2x)
<i>Multiply the first term</i>	$(3 + 7x) (6 + 2x) = 6 \cdot 3 = 18$ 18
<i>Multiply the outer terms</i>	$(3 + 7x) (6 + 2x) = 3 \cdot 2x = 6x$ 18 + 6x
<i>Multiply the inner terms</i>	$(3 + 7x) (6 + 2x) = 7x \cdot 6 = 42x$ 18 + 6x + 42x
<i>Multiply the last terms</i>	$(3 + 7x) (6 + 2x) = 7x \cdot 2x = 14x^2$ 18 + 6x + 42x + 14x ²
<i>Combine like terms</i>	$18 + 6x + 42x + 14x^2 = 18 + 48x + 14x^2$
Answer	18 + 48x + 14x²

5. Simplify Exponents of a number.

- See the EXPONENTS section for a review

PRIME FACTORIZATION**WAYS TO OBTAIN THE PRIME FACTOR**

- Repeatedly divide by prime numbers. A prime number is a positive integer greater than one that can only be divided by itself and one. Some examples are 2,3,5,7,11,13,17, and 19. 1 is NOT a prime number
- Choose any pair of factors and split these factors until all the factors are prime.
- Work backwards from the answers, seeing which one is BOTH only prime numbers, *and* produces the correct product

SAMPLE	What is the prime factorization for 68?
<i>Divide by 2 (a prime number)</i>	$68 \div 2 = 34$
<i>The correct way to represent prime factorization</i>	2×34
<i>Divide 34 by 2 and you are left with 17 (a prime number)</i>	$34 \div 2 = 17$
Answer	$2 \times 2 \times 17$

GREATEST COMMON FACTOR (GCF/GCD)

The **greatest common factor** is the largest integer that is a common factor of all the given integers.

FIND THE GCF BY:

- Finding the prime factorization of each integer.
- The GCF is the product of all prime factors common to every number.

SAMPLE	What is the greatest common factor of 8 and 44?
<i>Find the prime factorization of each integer</i>	$8 = 2 \times 2 \times 2$ $44 = 2 \times 2 \times 11$
<i>Identify the common prime factors</i>	$8 = 2 \times 2 \times 2$ $44 = 2 \times 2 \times 11$
<i>Multiply the common prime factors</i>	$2 \cdot 2 = 4$
Answer	4

LEAST COMMON DENOMINATOR (LCD/LCM) (sample on next page)

The **least common denominator** (multiple) is the smallest integer that is a common multiple (denominator) of the given integers.

FIND THE LCD BY:

- Finding the prime factorization of each integer.
- Take the greatest power on **each** prime and multiply them to obtain the LCD.

SAMPLE	What is the least common denominator of 12, 50, and 90?
<i>Find the prime factorization of each integer</i>	$12 = 2 \times 2 \times 3$ $50 = 2 \times 5 \times 5$ $90 = 2 \times 3 \times 3 \times 5$
<i>Identify the prime factor that appears most frequently within the prime factorization</i>	$12 = \mathbf{2} \times 2 \times 3$ $50 = 2 \times \mathbf{5} \times 5$ $90 = 2 \times \mathbf{3} \times 3 \times 5$
<i>List the most frequent prime factors with their exponent as the number of times it appeared</i>	12: 2^2 50: 5^2 90: 3^2
<i>Multiply the prime factors</i>	$2^2 \cdot 5^2 \cdot 3^2$
Answer	900

FACTORING

Factoring is writing a math expression as a product of factors. For example: writing 14 as (2)(7), where 2 and 7 are factors of 14. Factoring can also be done with trinomial and polynomial expressions.

- **Always factor as much as you can!** Often all terms in an expression have a common factor. FIRST group the like terms, and then find the greatest common factor and extract it (this is like the distributive law in reverse).

SAMPLE	Factor $5x + 7x$
<i>x is a factor of both $5x$ and $7x$, extract x and add the contents of the parentheses</i>	$x(5 + 7)$
Answer	$12x$

SAMPLE	Factor $14a + 7 + 21a$
<i>Organize like terms</i>	$14a + 21a + 7$
<i>Since a is a factor of both $14a$ and $21a$, extract a and add the content of the parentheses</i>	$a(14 + 21) + 7$
Answer	$35a + 7$

TRINOMIALS

- **Always factor as much as you can!** Often all terms in an expression have a common factor, first group the like terms and then find the greatest common factor and extract it (this is like the distributive law in reverse).
- Next Reverse the FOIL method to get the factored form:
 1. Set up a product of two expressions, where parentheses hold each of the two expressions.
 2. Find the factors that go in the first positions.
 3. Look at the signs before the second and third terms in the trinomial:
 - two negative signs (for example: $x^2 - 2x - 3$): the signs in each expression are opposite with the larger number being negative
 - two positive signs (for example: $x^2 + 4x + 3$): the signs are both positive
 - negative then a positive (for example: $x^2 - 4x + 3$): the signs are both negative
 - positive and negative (for example: $x^2 + 2x - 3$): the signs are opposite and the larger number is positive
 4. Find the factors that go in the last positions.
- Check your work

SAMPLE	Factor the trinomial: $x^2 - 4x - 32$
Reverse the FOIL method to get the factored form	$x^2 - 4x - 32$
x multiplied x equals x^2 thus one x in each parenthesis	$(x +/- _)(x +/- _)$
Since the signs on the 2 nd and 3 rd trinomial terms are both negative, the signs of the second term in each factor must be opposite	$(x - _)(x + _)$
What two numbers multiplied by one another would equal 32?	Possibilities: 32 and 1 16 and 2 8 and 4
Since the signs on the 2 nd and 3 rd trinomial terms are both negative, the larger multiple will have a negative sign	Possibilities: -32 and 1 -16 and 2 -8 and 4
Out of the possibilities, which pair added to one another equals -4?	-8 and 4
Plug these into the equation	$(x - 8)(x + 4)$
Check your work by using FOIL on the two factors	$x^2 - 8x + 4x - 32$
Simplify	$x^2 - 4x - 32$
Answer	$(x - 8)(x + 4)$

POLYNOMIALS STRATEGIES

- **Always factor as much as you can!** Often all terms in an expression have a common factor, first group the like terms and then find the greatest common factor and extract it (this is like the distributive law in reverse).
- Look for perfect squares:
 $a^2 + 2ab + b^2 = (a + b)^2$
 $a^2 - 2ab + b^2 = (a - b)^2$
- Look for the difference of squares:
 $a^2 - b^2 = (a + b)(a - b)$
- Others
 $(a + b) \times c = ac + bc$
 $(a - b) \times c = ac - bc$
 $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
 $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- Factor by trial and error
- Reverse the FOIL method to get the factored form:
 1. Set up a product of two expressions, where parentheses hold each of the two expressions.
 2. Find the factors that go in the first positions.
 3. Find the factors that go in the last positions.
- Check your work

SAMPLE	Factor $a^2 - 81$
Since $9^2 = 81$, this looks like a difference of squares	$(a + 9)(a - 9)$
Check your work by using FOIL	$a^2 + 9a - 9a - 81$
The positive and negative $9a$ cancel each other out	$a^2 - 81$
Answer	$(a + 9)(a - 9)$

SAMPLE	Factor $3x^2 + 9x + 6$
Factor out 3	$3(x^2 + 3x + 2)$
Factor by trial and error, since the second and third term in the parentheses are positive, the signs in each factor must be positive	$3(x + _)(x + _)$

What two numbers multiplied by one another would equal 2 and added to one another would equal 3? How about 2 and 1?	$3(x + 2)(x + 1)$
Check your work by using FOIL on the two factors in parentheses	$3(x^2 + 2x + 1x + 2)$
Simplify	$3(x^2 + 3x + 2)$
Distribute the 3	$3x^2 + 9x + 6$
Answer	$3(x + 2)(x + 1)$

SAMPLE ALGEBRA PROBLEMS

SAMPLE	Name the like terms in $7s + 9y + y$
Answer	$9y, y$

SAMPLE	Explain why $7a + 8z - 9x$ is in simplest form.
Answer	It has no like terms and no parentheses.

SAMPLE	Explain why $6 + 2(x - 4)$ is not in simplest form.
Answer	The two has not been distributed to the terms in the parentheses, and then simplified by combining like terms.

SAMPLE	Simplify $r + 3(s + 7r)$
Distribute the 3 to the contents of the parenthesis	$r + 3(s) + 3(7r) = r + 3s + 21r$
Organize like terms	$1r + 21r + 3s$
Combine like terms	$1r + 21r + 3s = 22r + 3s$
Answer	$22r + 3s$

SAMPLE	Simplify $8 + (-7)$
Adding a negative number is the same as subtracting that number	$8 - 7$
Answer	1

SAMPLE	If $14 = j - (-20)$, what is the value of j?
	$14 = j - (-20)$
Subtracting a negative number is the same as adding a positive number	$14 = j + 20$
Set all known values equal to j by subtracting 20 from both sides	$-20 + 14 = j + 20 - 20$
Answer	$-6 = j$

SAMPLE	How is the product $3 \cdot 3 \cdot 3$ expressed in exponential notation?
Set up the equation	$3^1 \cdot 3^1 \cdot 3^1$
When multiplying, add the exponents	3^{1+1+1}
Answer	3^3

SAMPLE	What is the value of $3t^5$ if $t = 2$?
Replace t with 2	$3(2^5)$
Simplify exponents	$3(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$
	$3(32)$
Answer	96

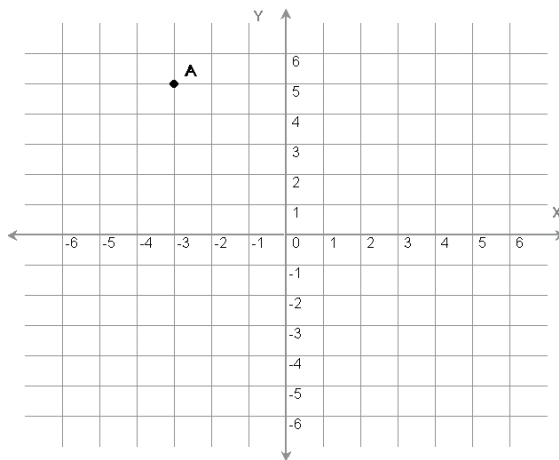
SAMPLE	Simplify $(-4a^5b)(8a^2)$
<i>Multiply like terms beginning with -4 and 8</i>	$-32 (a^5b)(a^2)$
<i>Multiply a^5 and a^2</i>	$-32 (a^7)(b)$
Answer	$-32a^7b$

SAMPLE	Solve $\frac{10}{y} = \frac{7}{y+3}$
<i>Cross multiply</i>	$10(y+3) = 7y$
<i>Solve for y, begin by distributing the 10</i>	$10y + 30 = 7y$
<i>Combine like terms, begin by subtracting 10y from both sides</i>	$-10y + 10y + 30 = 7y - 10y$
<i>After combining like terms</i>	$30 = -3y$
<i>Set all known values equal to y by dividing both sides by -3</i>	$30 \div -3 = -3y \div -3$
Answer	$-10 = y$

THE COORDINATE SYSTEM

GRID GRAPHS

The location of any point on a grid can be indicated by an **ordered pair** of numbers (x,y) where x represents the number of units on the horizontal line stemming away from zero (called the x-axis), and y represents the number of units on the vertical line stemming away from zero (called the y-axis). The x is always listed first, and the y is always listed second in an ordered pair. The numbers in an ordered pair are called **coordinates**. For example: if the x-coordinate is -3 and the y-coordinate is 5, the ordered pair for the point would be (-3,5).



SLOPE COORDINATES

- The **x-intercept** is the point where a line crosses the x-axis. It is found by setting $y = 0$ and solving the resulting equation.
- The **y-intercept** is the point where a line crosses the y-axis. It is found by setting $x = 0$ and solving the resulting equation.

SAMPLE	What are the coordinates of the x-intercept of the line $4y - x = 5$?
<i>Set up the equation</i>	$4y - x = 5$
<i>Set $y = 0$ and solve for x</i>	$4(0) - x = 5$
	$-x = 5$
<i>Multiply both sides by -1</i>	$(-1) \cdot x = (-1) 5$
	$x = -5$
Answer	$(-5, 0)$

GEOMETRY

BASICS

- The angles of any four sided figure always add up to 360°
- Two lines are **perpendicular** (\perp) when they meet at a 90° angle
- Two lines are **parallel** (\parallel) when they never intersect
- **Bisect** means to cut in half

SQUARES

- Each of the 4 sides are always equal in length
- Each of the 4 angles is always equal to 90°
- The area (A) of a square is found by squaring the measurement of one side
 - $A = s^2$
- Find the perimeter by adding up the length of all the sides
 - Perimeter = $4s$

RECTANGLES

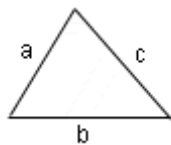
- Opposite sides are always equal
- Each of the 4 angles is always equal to 90°
- The area of a rectangle is found by multiplying the rectangle's length by its width
 - $A = lw$
- Find the perimeter by multiplying the length by two and the width by two and adding those products
 - Perimeter = $2l + 2w$

CIRCLES

- There are 360° in a circle
- Radius = distance from the center to any point on the edge of the circle (r)
- Diameter = straight line distance from one point on the circle to another, passing through the center point (d)
- $\pi = 3.14$ (π)
- The area of a circle is found by multiplying π by the radius squared
 - $A = \pi \cdot r^2$
- Circumference is the distance around the outside of the circle, find it by multiplying two by π by the radius
 - Circumference = $2 \pi \cdot r$

TRIANGLES

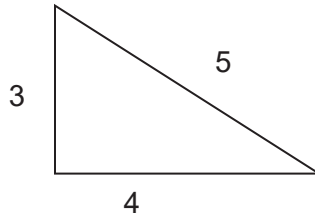
- Each of the 3 angles will always add up to 180°
- On right triangles two sides intersect to form a 90° angle
- The area of a triangle is found by multiplying the triangle's base by its height and dividing the product in half
 - $A = \frac{1}{2} bh$
- Find the perimeter by adding up the length of all the sides



- A hypotenuse is the side of a right triangle that is opposite the right (90°) angle. By using the Pythagorean Theorem one can find the length of an unknown side of a right triangle.
 - The Pythagorean Theorem is: $a^2 + b^2 = c^2$, where c equals the hypotenuse.

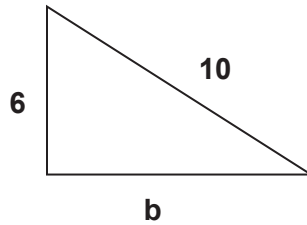
SAMPLE

In the right triangle below, the length of side $a = 3$, the length of side $b = 4$ and the hypotenuse (side c), has a length of 5. Using the Pythagorean Theorem ($a^2 + b^2 = c^2$), we see that $3^2 + 4^2 = 5^2$.



SAMPLE

Determine the length of side b , given that side $a = 6$ and side $c = 10$



<i>Use the Pythagorean Theorem</i>	$a^2 + b^2 = c^2$
<i>Plug in known values</i>	$6^2 + b^2 = 10^2$
<i>Combine like terms (subtract 6^2 from both sides)</i>	$b^2 = 10^2 - 6^2$
<i>Simplify exponents ($1 \cdot 10 \cdot 10$) - ($1 \cdot 6 \cdot 6$)</i>	$b^2 = 100 - 36$
	$b^2 = 64$
<i>Obtain the square root of 64 ($b^2 = 8^2$)</i>	$\sqrt{b^2} = \sqrt{64}$
Answer	$b = 8$